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Here neither $s^2=s$ nor $sx=xs$. If we say that in order to save whole these equations we may employ a different symbol for every application of the adjective small, how can we express the meaning which is common to them all, and in virtue of which the word small exists as an element of language?

Diffident as I am with respect to all these remarks on a method in which I find so much to admire, I am yet more so with respect to the following. But it seems to me that we cannot say that

$$x(1-x)=0$$

expresses *proprio vigore*, that is, in virtue of antecedent conventions, what is called the principle of contradiction.

In ordinary language we have words which, independently of this principle, express negation; we say red, not red, and the like; but in the 'Laws of Thought' there is no other means of expressing not red than by $1-x$, x denoting red.

Now the interpretation of this symbol $1-x$ seems to me to be given by the principle of contradiction, and therefore I should rather say that the equation $x(1-x)=0$ is interpreted by that principle than that it expresses it. In accordance with this view, the equation $x^2=x$ would appear to be independent of the principle of contradiction.

On Boole's 'Laws of Thought.' *By the* Rev. Robert Harley, *F.R.S.*

This paper was intended as a supplement to some "Remarks on Boole's Mathematical Analysis of Logic," which the author submitted to the Section at the Nottingham Meeting, an abstract of which was printed in the Report for 1866. (See Transactions of the Sections, pp. 3–6.)

From the logical equation $x^2=x$, the equation $x-x^2=0$ is derived by subtracting x^2 from both members, and the result is put under the form $x(1-x)=0$ by the law of distribution. It is to be observed, however, that at every step of the process the principle of identity $x=x$ is assumed, and in Boole's interpretation of the final result the same principle is used, for it is implied that the x without the brackets is identical with the x within. Further, in the final interpretation not only is the principle of contradiction (or non-contradiction) employed, as Leslie Ellis points out in the latter part of his 'Observations,' but the principle of excluded middle is also employed. For in interpreting $1-x$ to mean not-x, it is tacitly assumed that every one of the things of which the universe, represented by unity, is made up, is either x or not x. It would thus appear that these three principles, identity, contradiction, and excluded middle, are incapable of being reduced to more elementary truths. They are axiomatic, and Boole made use of them unconsciously in framing his laws of logical interpretation. ('Laws of Thought,' chap. iii. prop. iv.)

In chap. iii. § 5, Boole, by three different methods, one of which is partly logical, and the other two are wholly algebraical, deduces the equation

$$f(1)f(0)=0$$

from the equation for the expansion or development of any logical function $f(x)$, viz.

$$f(x)=f(1)x+f(0)(1-x),$$

where $f(x)$ may or may not involve other class symbols than x. The latter equation is established in chap. v. § 10, by means of the principle that it is lawful to treat x as a quantative symbol susceptible only of the values 0 and 1. But it is worthy of notice that the former equation may be directly established by means of the same principle. For, treating $f(x)=0$ as an algebraic equation, of which the root x has only the values 1 and 0, we have at once, by the theory of equations,

$$f(1)f(0)=0.$$

The influence of Boole's ideas may be traced in works apparently so diverse as Professor W. Stanley Jevons's 'Substitution of Similars,' Professor P. G. Tait's 'Quaternions,' and Sir Benjamin Brodie's 'Calculus of Chemical Operations.' The system of logic proposed by Mr. Jevons is closely analogous to, and in some respects identical with, that given by Boole; but it is distinguished from the latter by the rejection of the calculus of 1 and 0. In a little work entitled "Pure Logic, or the Logic of Quality apart from Quantity," Mr. Jevons has urged various objections to certain parts of Boole's system, more particularly to the numerical calculus. The author of this paper has briefly considered those objections in the concluding portion of an article on "Boole's Life and Writings," which he contributed to the July Number of the British Quarterly Review for 1866 (pp. 141–181).

Hamilton's theory of quaternions, as expounded by Prof. Tait, has its logical basis in principles which were first brought clearly to light in the course of Boole's remarkable inquiries. No one can read the earlier chapters of Prof. Tait's 'Quaternions,' and compare them with the earlier chapters of Boole's 'Laws of Thought,' without being struck with the similarity, not to say the identity, of many of the processes employed in both works. Treating of the properties of the quaternion symbols S, K, V, the expounder of Hamilton's system remarks, "It is curious to compare the properties of these quaternion symbols with the Elective Symbols of Logic, as given in Boole's wonderful treatise on the 'Laws of Thought,' and to think that the same grand science of mathematical analysis, by processes remarkably similar to each other, reveals to us truths in the science of *position* far beyond the powers of the geometer, and truths of deductive reasoning to which unaided thought could never have led the logician." (Tait's Quaternions, p. 50, footnote.)

Sir Benjamin Brodie has endeavoured to do for chemistry what Boole has done for logic,—to reduce it under the domain of mathematics, using the term "mathematics" in the enlarged sense, explained in the author's former communication. Of the validity of Sir Benjamin's proposed "method for the investigation, by means of symbols, of the laws of the distribution of weight in chemical change," it is not necessary to speak here. But that method is interesting, as being undoubtedly the first attempt to "free the science of chemistry from the trammels imposed upon it by accumulated hypotheses, and to endow it with the most necessary of all the instruments of progressive thought, an exact and rational language." Sir Benjamin's system was evidently suggested by Boole's 'Laws of Thought.' Whether the soil into which he has transplanted Boole's ideas be congenial or not, remains to be seen.

But the most remarkable amplification of Boole's conceptions which the author has hitherto met with is contained in a recent paper by Mr. C. S. Peirce, on the "Logic of Relatives" (Memoirs of the American Academy, vol. ix.). Mr. Peirce divides logical terms into three grand classes. "The first embraces those whose logical form involves only the conception of quality, and which therefore represent a thing simply, as 'a ——.' These discriminate objects in the most rudimentary way, which does not involve any consciousness of discrimination. They regard an object as it is in itself as *such* (quale); for example, as horse, tree, or man. These are *absolute terms*. The second class embraces terms whose logical form involves the conception of relation, and which require the addition of another term to complete the denotation. These discriminate objects with a distinct consciousness of discrimination. They regard an object as over against another, that is, as relative; as father of, lover of, or servant of. These are *simple relative terms*. The third class embraces terms whose logical form involves the conception of bringing things into relation, and which require the addition of more than one term to complete the denotation. They discriminate, not only with consciousness of discrimination, but with consciousness of its origin. They regard an object as medium or third between two others; that is, as conjugative, as giver of —— to ——, or buyer of —— for —— from ——. These may be termed *conjugative terms*." "Boole's logical algebra," says Mr. Peirce, "has such singular beauty, so far as it goes, that it is interesting to inquire whether it cannot be extended over the whole realm of formal logic, instead of being restricted to that simplest and least useful part of the subject, the logic of absolute terms, which, when he wrote, was the only formal logic known." The object of Mr. Peirce's paper is to show that this extension is possible. Some account was given of the notation and processes employed.

On Musical Intervals. *By* William Spottiswoode, *M.A., F.R.S.*
