MIND

A QUARTERLY REVIEW

OF

PSYCHOLOGY AND PHILOSOPHY.

EDITED BY

G. F. STOUT,

WITH THE CO-OPERATION OF PROFESSOR H. SIDGwick, PROFESSOR W. WALLACE,

DR. VENN, AND DR. WARD.

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as appendix) are the less to be overlooked, because here M. Picart does what he can, in other way than by the much-missed index, to bring together the multiplex threads of his whole inquiry. In the last paragraphs of all, there is a striking imagination of the state of the mind of an Idealist transported from the beginning of the century, when he worked so confidently for human progress and the end of the century was with its vast increase of scientific knowledge and increasing sense of the limits set to positive science and its ever-growing burden of social difficulties and perils. The Idealist, it is allowed, would have to abate much of his practical optimism, and it would be no longer deal so lightly as he did with philosophical questions because they had failed of success. None the less he might truly claim to have done a real stroke of work by his day. He had broken ground in every one of the lines upon which psychology has since advanced—an effort only partially recognised in the foregoing notice but admirably shown in the book itself. He had also had his own measure of philosophic insight when he proclaimed that all other human search and all human striving should own the sway of a science of "Ideas".

G. ChoM RoBERTON.


The appearance of the first volume—a very bulky one—of Dr. Schröder’s great work marks an important stage in the progress of Exact Logic. With the exception of the brief former paper of the same writer (Der Operationskreis des Logikkalkuls) the subject has hitherto received no presentation in Germany; and, for the purpose of making it accessible to the reader who approaches it for the first time, this presentation is practically the only thing that yet exists in any language.

Mr. Charles S. Peirce, to whom Symbolic Logic owes its present state of development, wrote his papers with the brevity and abstractness that befit a scientific journal. Dr. Schröder’s book will be objected to on the ground that it is unnecessarily diffuse; but it should be remembered that the subject has had hard work to get itself recognised, and that it is a principle of psychology that a certain degree of volubility in a sensation is essential to the producing of a lasting impression. It must be admitted that the book is discursive to the last degree. On the other hand, it is not undesirable that everything that can be said, by way of elucidation and reinforcement, should once be said; coming books can be written with all the greater conciseness. It goes without saying that Dr. Schröder’s book is a work of true German thoroughness, and patience with teasing details; it will be impossible hereafter for any one to write upon the subject without having made himself familiar with the views set forth in this volume.

The plan of Dr. Schröder in his book follows closely upon that of Mr. Peirce as set forth in the American Journal of Mathematics; that is to say, all the formulae are established by analytical proofs based upon the definitions of sum, of product, and of the negative, and upon the axioms of identity and that of the syllogism. (Later, it is found necessary to add another axiom to cover one of the two parts of the distribution law.) The proofs are, for the most part, the same as those given by Prof. Peirce; but frequently alternative proofs are given in addition, and occasionally the method of treatment varies. Dr. Schröder considers it an important difference between his treatment and that of Mr. Peirce that with him (in this first volume) the letters stand for classes (p. 290), while with Mr. Peirce they stand for statements. This is not a strictly correct account of Mr. Peirce’s treatment. The great effect which that writer has had at once simplifying and extending the whole body of logical doctrine (not merely its symbolic exposition) is based upon his identification of the proposition with the relation of values. It is plain that (provided universal propositions are taken as not implying the existence of their terms) there is no difference between

The statement $P$ implies the statement $P$, or, if $P$ then $P$, and the term $t$ implies the term $t$, or, every $t$ is a $t$,

as far as the part they can play in a logical structure is concerned. The relation between $P$ and $P$, and the relation between $t$ and $t$, are both sufficiently defined by saying that they are transitive relations, in the sense in which the term is used by De Morgan; that is (if we use a common sign $\leq$ to express this common relation), we shall have for a (daily) definition of the relation

$s \leq p$,

(whether $s$ and $p$ stand for terms or for propositions), whatever $p$ is, that $s$ shall also be; or, whatever is $s$, that shall also be $p$. Expressed symbolically, this will be

$s \leq p$ is-the-same-thing-as

$(p \leq x) \leq (s \leq x)$ D,

and is-the-same-thing-as

$(x \leq s) \leq (x \leq p)$ D,

where $x$ stands for anything whatever. This is, as it happens, in strict accordance with Mill’s account of the proposition; he says (Logic, eighth edition, p. 155) that it asserts that “all things which have a certain attribute have along with it a certain other
in his second volume (the advance sheets of part of which lie before us), develops the transitive relation for the dictum of "if $x < y$, then $y < z$". This is a generalization of the relation of "less than" to "less than". He considers that

$$x < y + z$$

is of a different content, according to whether the letters stand for terms or for propositions. It is true that if $y$ or else $z$ is said to be a logical consequence of $x$, then the logical consequence of $x$ is either always $y$ or always $z$ (or both); and it is also true that, on the other hand, "men are either honest or else unhappy" is satisfied by some individuals being honest and other individuals being unhappy. But so also any material propositional sequence, such as "if it rains, either I stay in or else I take an umbrella," is satisfied by some instances of its being followed by my staying in and all other instances being followed by my taking an umbrella. Dr. Schröder in fact, seems to pay too little attention to material following. Logical following has its exact parallel in the proposition in the case of the singular subject. She is either a queen or a fairy" does not admit of part of her being a queen and part of her being a fairy." There seems, in fact, to be a close relationship between the logical sequence between propositions, and the sequence between terms when the subject is singular. Again, Dr. Schröder, after showing that, for propositions,

$$\frac{(a < b)}{a + b}$$

that is, that

- "If some are wise, some will be unfortunate" is equivalent to

- "Either all are wise, or else some are unfortunate," asks, what could be the meaning of this if $a$ and $b$ stood for terms instead of for propositions? The answer is very easy. The last sentence is an abbreviated form—made possible by the accidents of language (see my paper in "Some Characteristics of Symbolic Logic," Am. Jour. Psychology, 1889)—for the complete statement,

"The possible" implies that, all are wise or else that some are unfortunate." That is, the full expression for the equation written above is—
CRITICAL NOTICES.

(a < b) = (x < a + b).

When a and b are terms, this is—

‘All a is b’ is the same thing as ‘everything is either not-a or else b’.

a transformation which is as valid and as simple for terms as it is for propositions.

In his treatment of the signification of the negative term,—a subject upon which very many logicians have gone astray,—Dr. Schröder virtually sets forth the correct doctrine (for instance, on p. 337), but not with quite sufficient constancy or clearness. It is true that there is not much difference between the presence of a quality and the absence of a quality, and hence that the signification of a negative term is of very much the same nature as that of a positive term, so long as the quality which marks its signification is one and indivisible. It makes no difference whether we divide numbers up into even and not-even or into odd and not-odd. But the case is very different when we come to complex qualities. We may set forth symbolically the two-fold force of a term in the following fashion: Since the aggregate of objects to which it applies is of the nature of a logical sum, and the congeries of qualities which it implies is of the nature of a logical product, the full import of a term, as civilisation, ɛ, will be—

ɛ = (ɛ₁ + ɛ₂ + · · · · · · +) γ₁ γ₂ γ₃ · · · · · ·

where ɛ₁, ɛ₂, · · · · · · stand for all the different instances of its application (as the civilisation of the Assyrians, that of the Greeks, and so on), and γ₁, γ₂, γ₃ · · · · · · stand for all the elements which are essential to its signification (as, being in the possession of good laws, ensuring the safety of the person and of property, securing a certain amount of happiness to a considerable number of individuals, &c.), and where each one of the instances has all of the essential qualities attached to it. What will then be the negative of the term civilisation? It will be, in accordance with the usual rule for taking the negative—

ɛ' = ɛ₁ ɛ₂ ɛ₃ · · · · · · (γ₁ + γ₂ + γ₃ · · · · · ·),

that is, the non-civilisations are, at once, not any one of the civilisations, and at the same time they have the quality of being deficient in some one, at least, of the qualities that are essential to a thing’s being a civilisation (the qualities, that is, in the absence of any one of which we should refuse to apply the name). The intent of the positive term and of the negative term are therefore extremely different; the one involves a combination of quality-elements, the other an alternation of absences of quality-elements. It is only in the case of terms of indivisible intent (as hot, cold, blue, heavy, parallel) that the difference between them becomes insignificant. When, therefore,
duced by one half,—a single statement with, this copula is the representative of a statement together with its dual opposite in terms of the other copula. These are advantages which are possessed by both of the symmetrical copulas, 'no A is B' and 'all but A is B'"; and by neither of the unsymmetrical copulas, 'all A is B' and 'none but A is B'. 'All A is B' has, of course, a great superiority in point of naturalness, but the others ought not to be treated as if they were non-existent.

When it comes to the solution of problems, Dr. Schröder discards altogether Mr. Peirce's method, which consists in a consistent carrying out of the properties of the copula ≤, for the far simpler method of first reducing the second member of the statement to "zero," or "non-existent,"—that is, of transposing all the terms into the first member of the statement. His treatment of this part of his subject could not be improved upon.

A number of interesting points we have left ourselves no room to speak about. Dr. Schröder proves that subtraction and division are inexecutable operations, and that the words are pure nonsense-words in Logic. He also shows that only an historical interest attaches to the labours of Boole in the field of symbolic Logic. A particularly interesting passage is that in which he proves that the second subsumption of the distribution-law, viz.,

\[ a \cdot (b + c) \leq ab + ac, \]

cannot be deduced from the other axioms and the definitions, by showing that in the logical calculus of groups all these other axioms and definitions hold but that this subsumption is not true. Into that calculus, however, the idea of the negative does not enter; hence it is only proved that the above subsumption cannot be deduced from the axioms and definitions exclusive of the definition of the negative.

CHRISTINE LADD FRANKLIN.


In spite of all that has been written about Spinoza, the authors of this work have contrived to say something new. There are important differences in the theory of knowledge as set forth in Spinoza's successive works—the Short Treatise, the De Intellectus Emendatione, and the Ethics—and, even in its final form, it is held to be far from clear by most of those who have expressly examined it. I know of no other discussion which can compare

1 It is virtually in terms of this copula that Mr. Mitchell has developed his Algebra of Logic.