the following considerations to show that this interpretation of history is not exactly true. In the first place, it is said that a distinction was taken between these attributes, as though they were previously confused. Now there is not the least evidence of this. A German logician, has, indeed, by a subtle misconception, considered extension as a species of comprehension, but, to a mind beginning to reflect, no notions seem more unlike. The mental achievement has been the bringing of them into relation to one another, and the conception of them as factors of the import of a term, and not the separation of them. In the second place it is correctly said that the doctrine taught by the Port Royalists is substantially contained in the work of a Greek commentator. That work is no other than Porphyry’s Isagoge; and therefore it would be most surprising if the doctrine had been totally overlooked by the schoolmen, for whether their acuteness was as marvellous as Hamilton taught or not, they certainly studied the commentary in question as diligently as they did the Bible. It would seem, indeed, that the tree of Porphyry involves the whole doctrine of extension and comprehension except the names. Nor were the scholastics without names for these quantities. The *partes subjectivae* and *partes essentiales* are frequently opposed; and several other synonyms are mentioned by the Conimbricenses. It is admitted that Porphyry fully enunciates the doctrine; it must also be admitted that the passage in question is fully dealt with and correctly explained by the medieval commentators. The most that can be said, therefore, is that the doctrine of extension and comprehension was not a prominent one in the mediæval logic.†

---

* Porphyry appears to refer to the doctrine as an ancient one.
† The author of “De Generibus et Specierbus” opposes the *integral* and *distinctive* wholes. John of Salisbury refers to the distinction of comprehension and extension, as something “quod fore in omnium ore celebrem, et alius scilicet esse quod appellativa *significant*, et alius esse quod *nominat*. Nominatur singularia, sed universalia *significantur*.” (Metalegismus, lib. 2, cap. 20. Ed. of 1620, p. 111.)

Vincentius Lullioscensis (Spectrum Doctrinale, Lib. III. cap. xi.) has the following: “Si vero queritur gram hoc universale *homoe* sit in qualibet homine secundum se totum esse secundum partem, dicendum est quot secundum se totum, il est secundum quoniam et partem distinctas, . . . Non autem secundum quantulum partem subjectivam.” William of Auvergne (Præstis Chronica, Vol. III. p. 77) speaks of “totalitatem tamen, quæ est ex partibus rationis seu distinctionibus, et hoc partes sunt genus et differentia; alio modo partes speciei individuum sunt, quum ipsum speciem, cum de eis praedicatur, sibi invicem quodammodo partientur.”

---

* This is quoted from Bainers (Port Royal Logic, 2d ed. p. xxxiii.), who says that he is indebted to Sir William Hamilton for the information.
A like degree of historical error is commonly committed in reference to another point which will come to be treated of in this paper, and at least, as it is most intimately with the subject of comprehension and extension, inasmuch as it also is founded on a conception of a term as a whole composed of parts, I mean the distinction of clear and distinct. Hamilton tells us that we owe the discrimination to the neatness of the great Leibnitz. By the Cartesian the distinction had not been taken; though the authors of the Port Royal Logic quite agree that we may well marvel how they failed explicitly to enunciate it. (Lectures on Logic: Lecture IX.) Now, in fact, all that the Port Royalists say about this matter is copied from Descartes, and their variations from his wording serve only to confuse what in him is tolerably distinct. As for Leibnitz, he himself expressly avows that the distinction drawn by Descartes is the same as his own. Nevertheless, it is very much more clear with Leibnitz than with Descartes. A philosophical distinction emerges gradually into consciousness; there is no moment in history before which it is altogether unrecognized, and after which it is perfectly luminous. Before Descartes, the distinction of confused and distinct had been thoroughly developed, but the difference between distinctness and clearness is uniformly overlooked. Scotus distinguishes between conceiving confusedly and conceiving the confused, and since any obscure concept necessarily includes more of the proper object, there is always in what is obscurely conceived a conception of something confused; but the schoolmen came no nearer than this to the distinction of Descartes and Leibnitz.

§ 2. Of the Different Terms applied to the Quantities of Extension and Comprehension.

Extension and comprehension are the terms employed by the Port Royalists. Owing to the influence of Hamilton, universal is now frequently used for comprehension; but it is liable to be confounded with intensity, and therefore is an objectionable word. It is derived from the use of cognate words by Cajetan and other early writers. External and internal quantity are the terms used by many early Kantians.

If we were to go to later authors, the examples would be endless. See any commentary in Phys. Lib. I.

* Part I. chap. ix.
† Principia, Part I. 48 et seq.
‡ Eighth Letter to Burnet.
§ 3. Of the Different Senses in which the Terms Extension and Comprehension have been accepted.

The terms extension and comprehension, and their synonyms, are taken in different senses by different writers. This is partly owing to the fact that while most writers speak only of the extension of concepts, others apply these terms equally to concepts and judgments (Rößling), others to any mental representation (Überweg and many French writers), others to cognition generally (Baumgarten), others to “terms” (Fowler, Spalding), others to names (Sheffer), others to words (McCregor), others to “meanings” (Jevons), while one writer speaks only of the extension of classes and the comprehension of attributes (De Morgan in his Syllabus).

Comprehension is defined by the Port Royalists as “those attributes which an idea involves in itself, and which cannot be taken away from it without destroying it.”

It will be remembered that the marks of a term are divided by logicians first into the necessary and the accidental, and that those necessary marks are subdivided into such as are strictly essential, that is, contained in the definition, and such as are called proper. Thus it is an essential mark of a triangle to have three sides; it is a proper mark to have its three angles equal to two right angles; and it is an accidental mark to be treated of by Euclid.

The definition of the Port Royalists, therefore, makes comprehension include all necessary marks, whether essential or proper.

The Port Royalists attribute comprehension immediately to any ideas. Very many logicians attribute it immediately only to concepts. Now a concept, as defined by them, is strictly only the essence of an idea; they ought therefore to include in the comprehension only the essential marks of a concept. These logicians, however, abstract so entirely from the real world, that it is difficult to see why these essential marks are not at the same time all the marks of the object as they suppose it.

There can, I think, be no doubt that such writers as Gerlach and Sigwart make comprehension include all marks, necessary or accidental, which are universally predictable of the object of the concept.

Again, most German writers regard the comprehension as a sum either of concepts (Drobisch, Bachmann, etc.) or of elements of intuition (Trendelenburg). But many English writers regard it as the
PROCEEDINGS OF THE AMERICAN ACADEMY

Again, logicians differ as to whether by extension they mean the concepts, species, things, or representations to which the term is habitually applied in the judgment, or all to which it is truly applicable. The latter position is held by Herbart, Kiesewetter, etc.; the former by Duncan, Spalding, Vorländer, Uebervogel, etc.

Some logicians include only actual things, representations, etc., under extension (Bachmann, Fries, Herbart); others extend it to such as are merely possible (Esser, Ritter, Gerlach).

Finally, some few logicians speak of the two quantities as numerical, while most writers regard them as mere aggregates of diverse objects or marks.


Until lately the law of the inverse proportionality of extension and comprehension was universally admitted. It is now questioned on various grounds.

Drobisch says that the comprehension varies arithmetically, while the extension varies geometrically. This is true, in one sense.

Lotze, after remarking that the only conception of a universal which we can have is the power of imagining singulars under it, urges that the possibility of determining a concept in a way corresponding to each particular under it is a mark of that concept, and that therefore the narrower concepts have as many marks as the wider ones. But, I reply, these marks belong to the concept in its second intention, and are not common marks of those things to which it applies, and are therefore no part of the comprehension. They are, in fact, the very marks which constitute the extension. No one ever denied that extension is a mark of a concept; only it is a certain mark of second intention.

Vorländer's objection is much more to the purpose. It is that if from any determinate notion, as that of Napoleon, we abstract all marks, all determination, what remains is merely the conception something, which has no more extension than Napoleon. "Something" has no uncertain sphere, meaning either this thing or that or the other, but has no general extension, since it means one thing only. Thus, before a race, we can say that some horse will win, meaning this one, that one, or that one; but by some horse we mean but one, and it therefore has no more extension than would a term definitely indicating which, although this latter would be more determinate, that is, would have

* I adopt the admirable distinction of Scotti between actual, habitual, and virtual cognition.
more comprehension. I am not aware that those who adhere to Kant's unmodified doctrine have succeeded in answering this objection.

Ubbeweg has the following remarks.* "To the higher representation, since conformably to its definition it contains only the common elements of content of several lower representations, belongs in comparison to each of the lower a more limited content, but a wider circuit. The lower representation, on the contrary, has a richer content but narrower circuit. Yet by no means by every diminution of increase of a given content does the circuit increase or diminish, nor by every increase or diminution of a given circuit does the content diminish or increase." I am surprised that he does not explain himself further upon this point, which it is the principal object of this paper to develop.

De Morgan says:† "According to such statements as I have seen, 'man residing in Europe, drawing breath north of the equator, seeing the sun rise before those in America,' would be a more intensively quantified notion than 'man residing in Europe'; but certainly not less extensive, for the third and fourth elements of the notion must belong to those men to whom the first and second belong." Mr. De Morgan adopts the definitions of extension and comprehension given by the Port Royalists. According to those definitions, if the third and fourth elements necessarily belong to the notion by which the first and second belong, they are parts of the comprehension of that second notion which is composed of the first and second elements, and therefore the two notions are equal in comprehension; but if this is not the case, then the second notion can be predicated of subjects of which the first cannot, for example, of 'man residing in Europe drawing breath south of the Equator'; for that there is really no such man will not affect the truth of the proposition, and therefore the second notion is more extensive than the first.

Two logicians, only, as far as I remember, Archbishop Thomson and Dr. W. D. Wilson,§ while apparently admitting Kant's law, wish to establish a third quantity of concepts. Neither gentleman has defined his third quantity, nor has stated what its relations to the other two are. Thomson calls his Denomination. It seems to be the same as Extension regarded in a particular way. Dr. Wilson terms his new quantity Preception; it has something to do with time, and appears to be generally independent of the other two. It is plain, indeed, that as

* Logik, 2v Aufl. § 54.
† Laws of Thought, 4th ed., §§ 52, 80.
‡ Formal Logic, p. 234. His doctrine is different in the Syllabus.
§ Logic, Part I. chap. II. § 5.

§ 4. Three Principal Senses in which Comprehension and Extension will be taken in this Paper.

I shall adopt Hamilton's terms, breadth and depth, for extension and comprehension respectively, and shall employ them in different senses, which I shall distinguish by different adjectives.

By the informed breadth of a term, I mean all the real-things of which it is predicable, with logical truth on the whole in a supposed state of information. By the phrase "on-the-whole" I mean to indicate that all the information at hand must be taken into account, and that those things of which there is not on the whole reason to believe that a term is truly predicative are not to be reckoned as parts of its breadth.

If T be a term which is predicable only of S, S', and S'', then the S's, the S''s, and the S'''s, will constitute the informed breadth of T. If at the same time, S' and S'' are the subjects of which alone another term T' can be predicated, and if it is not known that all S'''s are either S' or S'', then T is said to have a greater informed breadth than T'. If the S'''s are known not to be all among the S' s and S'' s, this excess of breadth may be termed certain, and, if this is not known, it may be termed doubtful. If there are known to be S''''s, not known to be S's or S''s, T is said to have a greater actual breadth than T'; but if no S'''s are known except such as are known to be S' s, and S'' s (though there may be others), T is to have a greater potential breadth than T'. If T and T' are conceptions in different minds, or in different states of the same mind, and it is known to the mind which conceives T that every S''' is either S' or S'', then T is said to be more extensively distinct than T'.

By the informed depth of a term, I mean all the real characters (in contradistinction to mere names) which can be predicated of it † (with

* For the distinction of extensive and comprehensive distinctness, see Scottus, l, dist. 2, qu. 3.
† That is, of whatever things it is applicable to.
logical truth, on the whole) in a supposed state of information; no character being counted twice over knowingly in the supposed state of information. The depth, like the breadth, may be certain or doubtful, actual or potential, and there is a comprehensive distinctness corresponding to extensive distinctness.

The informed breadth and depth suppose a state of information which lies somewhere between two imaginary extremes. These are, first, the state in which no fact would be known, but only the meaning of terms; and, second, the state in which the information would amount to an absolute intuition of all there is, so that the things we should know would be the very substances themselves, and the qualities we should know would be the very concrete forms themselves. This suggests two other sorts of breadth and depth corresponding to these two states of information, and which I shall term respectively the essential and the substantial breadth and depth.

By the essential depth of a term, then, I mean the really conceivable qualities predicated of it in its definition.

The defined term will not perhaps be applicable to any real objects whatever. Let, for example, the definition of the term T be this,

\[ T = P \lor P' \lor P''. \]

Then this sums up its whole meaning; and, as it may not be known that there is any such thing as \( P' \), the meaning of T does not imply that it exists. On the other hand, we know that neither \( P \), \( P' \), nor \( P'' \) is coextensive with the whole sphere of being. For they are determinate qualities, and it is the very meaning of being that it is indeterminate, that is, is more extensive than any determinate term. In fact, \( P \), for example, is a real notion which we never could have except by means of its contrast to something else. Hence we must know that

Whatever is not-\( P \) is not-T,

Whatever is not-\( P' \) is not-T,

and

Whatever is not-\( P'' \) is not-T.

Thus if we define the essential breadth of a term as those real things of which, according to its very meaning, a term is predicable, not-T has an essential breadth. We may therefore divide all terms into two classes, the essentially affirmative or positive and the essentially negative; of which the former have essential breadth, but no essential depth, and the latter essential breadth, but no essential depth. It must be noted, however, that this division is not the same as the similar one which language makes. For example, being, according to this, is an essentially negative term, inasmuch as it means that which can be predicated of whatever you please, and so has an essential breadth; while nothing is an essentially positive term, inasmuch as it means that of which you are at liberty to predicable what you please, and therefore has an essential depth. The essential subjects of being cannot be enumerated, nor the essential predicates of nothing.

In essential breadth or depth, no two terms can be equal; for, were that the case, the two terms would have the same meaning, and therefore, for logical purposes, would be the same term. Two terms may have unknown relations in these quantities, on account of one or other of them not being distinctly conceived.

Substantial breadth is the aggregate of real substances of which alone a term is predicable with absolute truth. Substantial depth is the real concrete form which belongs to everything of which a term is predicable with absolute truth.

General terms denote several things. Each of these things has in itself no qualities, but only a certain concrete form which belongs to itself alone. This was one of the points brought out in the controversy in reference to the nature of universals. As Sir William Hamilton says, not even the humanity of Linnean belongs to Newton, but a different humanity. It is only by abstraction, by an oversight, that two things can be said to have common characters. Hence, a general term has no substantial depth. On the other hand, particular terms, while they have substantial depth, inasmuch as each of the things, one or other of which are predicated of them, has a concrete form, yet have no substantial breadth, inasmuch as there is no aggregate of things to which alone they are applicable. In order to place this matter in a clearer light, I must remark, that I, in common with most logicians, take the copula in the sense of a sign of attribution, and not, like Hamilton, in the sense of a sign of equality in extension or comprehension. He exposes the proposition, "man is an animal," thus:

\[ \text{The extension of man} \quad \ldots \ldots \quad \text{Subject.} \]

\[ \text{equals} \quad \ldots \ldots \quad \text{Copula.} \]

\[ \text{a part or all of the extension of animal} \quad \ldots \ldots \quad \text{Predicate.} \]

*See, for example, De Generibus et Specificis, p. 548.
And thus he makes the predicate particular. Others interpret it thus:

\[
\begin{align*}
\text{Every man} \quad \text{Subject.} \\
\text{has all the attributes common to} \quad \text{Copula.} \\
\text{every animal} \quad \text{Predicate.}
\end{align*}
\]

It is in this latter sense that the copula is considered in this paper. Now, a particular is, as has been said, an alternative subject. Thus, “Some S is M” means, if S′, S″, and S‴ are the singular S′s, that either S′, or else S″, or else S‴, has all the attributes belonging to M.” A particular term, then, has a substantial depth, because it may have a predicate which is absolutely concrete, as in the proposition, “Some man is Napoleon.” But if we put the particular into the predicate we have such a proposition as this: “M has all the attributes belonging to S′, or else all those belonging to S″, or else all those belonging to S‴.” And this can never be true unless M is a single individual. Now a single individual substance is, I will not say an atom, but the smallest part of an atom, that is, nothing at all. So that a particular can have no substantial breadth. Now take the universal term “S.” We can say, “Any S is M,” but not if M is a real concrete quality. We cannot say, for instance, “Any man is Napoleon.” On the other hand, we can say “Any M is S,” even if M is a real substance or aggregate of substance. Hence a universal term has no substantial depth, but has substantial breadth. We may therefore divide all terms into substantial universals and substantial particulars.

Two terms may be equal in their substantial breadth and depth, and differ in their essential breadth and depth. But two terms cannot have relations of substantial breadth and depth which are unknown in the state of information supposed, because in that state of information everything is known.

In informed breadth and depth, two terms may be equal, and may have unknown relations. Any term, affirmative or negative, universal or particular, may have informed breadth or depth.

§ 5. The Conceptions of Quality, Relation, and Representation, applied to this Subject.

In a paper presented to the Academy last May, I endeavored to show that the three conceptions of reference to a ground, reference to a correlate, and references to an interpretant, are those of which logic must principally make use. I there also introduced the term “symbol,” to include both concept and word. Logic treats of the reference of symbols in general to their objects. A symbol, in its reference to its object, has a triple reference:

1st., Its direct reference to its object, or the real things which it represents;
2nd., Its reference to its ground through its object, or the common characters of those objects;
3rd., Its reference to its interpretant through its object, or all the facts known about its object.

What are thus referred to, so far as they are known, are:

1st., The informed breadth of the symbol;
2nd., The informed depth of the symbol;
3rd., The sum of synthetical propositions in which the symbol is subject or predicate, or the information concerning the symbol.

By breadth and depth, without an adjective, I shall hereafter mean the informed breadth and depth.

It is plain that the breadth and depth of a symbol, so far as they are not essential, measure the information concerning it, that is, the synthetical propositions of which it is subject or predicate. This follows directly from the definitions of breadth, depth, and information. Hence it follows:

1st., That, as long as the information remains constant, the greater the breadth, the less the depth;
2nd., That every increase of information is accompanied by an increase in depth or breadth, independent of the other quantity;
3rd., That, when there is no information, there is either no depth or no breadth, and conversely.

These are the true and obvious relations of breadth and depth. They will be naturally suggested if we term the information the area, and write

\[
\text{Breadth} \times \text{Depth} = \text{Area}.
\]

If we learn that S is P, then, as a general rule, the depth of S is increased without any decrease of breadth, and the breadth of P is increased without any decrease of depth. Either increase may be certain or doubtful.
It may be the case that either or both of these increases does not take place. If $P$ is a negative term, it may have no depth, and therefore adds nothing to the depth of $S$. If $S$ is a particular term, it may have no breadth, and then adds nothing to the breadth of $P$. This latter case often occurs in metaphysics, and, on account of not-$P$ as well as not-$P$ being predicated of $S$, gives rise to an appearance of contradiction where there really is none; for, as a contradiction consists in giving to contradictory terms some breadth in common, it follows that, if the common subject of which they are predicated has no real breadth, there is only a verbal, and not a real contradiction. It is not really contradictory, for example, to say that a boundary is both within and without what it bounds. There is also another important case in which we may learn that "$S$ is $P$," without thereby adding to the depth of $S$ or the breadth of $P$. This is when, in the very same act by which we learn that $S$ is $P$, we also learn that $P$ was covertly contained in the previous depth of $S$, and that consequently $S$ was a part of the previous breadth of $P$. In this case, $P$ gains in extensive distinctness and $S$ in comprehensive distinctness.

We are now in condition to examine Voltaire's objection to the inverse proportionality of extension and comprehension. He requires us to think away from an object all its qualities, but not, of course, by thinking it to be without those qualities, that is, by denying those qualities of it in thought. How then? Only by supposing ourselves to be ignorant whether it has qualities or not, that is, by diminishing the supposed information; in which case, as we have seen, the depth can be diminished without increasing the breadth. In the same manner we can suppose ourselves to be ignorant whether any American but one exists, and so diminish the breadth without increasing the depth.

It is only by confusing a movement which is accompanied with a change of information with one which is not so, that people can confound generalization, induction, and abstraction. **Generalization** is an increase of breadth and a decrease of depth, without change of information. **Induction** is a certain increase of breadth without a change of depth, by an increase of believed information. **Abstraction** is a decrease of depth without any change of breadth, by a decrease of conceived information. **Specification** is commonly used (I should say unfortunately) for an increase of depth without any change of breadth, by an increase of asserted information. **Supposition** is used for the same process when there is only a conceived increase of information.

"Determination, for any increase of depth. Restriction, for any decrease of breadth; but more particularly without change of depth, by a supposed decrease of information. Descent, for a decrease of breadth and increase of depth, without change of information."

Let us next consider the effect of the different kinds of reasoning upon the breadth, depth, and area of the two terms of the conclusion.

In the case of deductive reasoning it would be easy to show, were it necessary, that there is only an increase of the extensive distinctness of the major, and of the comprehensive distinctness of the minor, without any change in information. Of course, when the conclusion is negative or particular, even this may not be effected.

Induction requires more attention. Let us take the following example:

$S, S', S'', S'''$ have been taken at random from among the $M$'s; $S, S', S'', S'''$ are $P$:

... any $M$ is $P$.

We have here, usually, an increase of information. $M$ receives an increase of depth, $P$ of breadth. There is, however, a difference between these two increases. A new predicate is actually added to $M$; one which may or may be true, have been covertly predicated of it before, but which is now actually brought to light. On the other hand, $P$ is not yet found to apply to anything but $S, S', S'', S'''$, but only to apply to whatever else may hereafter be found to be contained under $M$. The induction itself does not make known any such thing. Now take the following example of hypothesis:

$M$ is, for instance, $P, P', P'', P'''$;

$S$ is $P, P', P'', P'''$;

... $S$ is all that $M$ is.

Here again there is an increase of information, if we suppose the premises to represent the state of information before the inferences. $S$ receives an addition to its depth; but only a potential one, since there is nothing to show that the $M$'s have any common characters besides $P, P', P'', P'''$. $M$, on the other hand, receives an actual increase of breadth in $S$, although, perhaps, only a doubtful one. There is, therefore, this important difference between induction and hypothesis, that the former potentially increases the breadth of one term, and actually increases the depth of another, while the latter potentially in-
creases the depth of one term, and actually increases the breadth of another.

Let us now consider reasoning from definition to definitum, and also the argument from enumeration. A defining proposition has a meaning. It is not, therefore, a merely identical proposition, but there is a difference between the definition and the definitum. According to the received doctrine, this difference consists wholly in the fact that the definition is distinct, while the definitum is confused. But I think there is another difference. The definitum implies the character of being designated by a word, while the definition, previously to the formation of the word, does not. Thus, the definitum exceeds the definition in depth, although only verbally. In the same way, any unanalyzed notion carries with it a feeling—a constitutional word—which its analysis does not. If this be so, the definition is the predicate and the definitum the subject, of the defining proposition, and this last cannot be simply converted. In fact, the defining proposition affirms that whatever a certain name is applied to is supposed to have such and such characters; but it does not strictly follow from this, that whatever has such and such characters is actually called by that name, although it certainly might be so called. Hence, in reasoning from definition to definitum, there is a verbal increase of depth, and an actual increase of extensive distinctness (which is analogous to breadth). The increase of depth being merely verbal, there is no possibility of error in this procedure. Nevertheless, it seems to me proper, rather to consider this argument as a special modification of hypothesis than as a deduction, such as is reasoning from definitum to definition. A similar line of thought would show that, in the argument from enumeration, there is a verbal increase of breadth, and an actual increase of depth, or rather of comprehensive distinctness, and that therefore it is proper to consider this (as most logicians have done) as a kind of inductive induction. These species of hypothesis and induction are, in fact, merely hypotheses and inductions from the essential parts to the essential whole; this sort of reasoning from parts to whole being demonstrative. On the other hand, reasoning from the substantial parts to the substantial whole is not even a probable argument. No ultimate part of matter fills space, but it does not follow that no matter fills space.