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The Common Sense of the Exact Sciences.

By the late William Kingdon Clifford. New York: Appletons. [International Scientific Series]

Attributed to Peirce by Fisch in *First Supplement* (internal evidence: the reference to F. E. Abbot's concept of space). Also, Peirce was personally acquainted with W. K. Clifford. This piece is unassigned in Haskell's *Index to* The Nation, vol. 1.

William Kingdon Clifford (1845-1879) was an English mathematician and philosopher. He was appointed professor of applied mathematics at University College, London, in 1870, and while there, was elected to the membership of the Metaphysical Society and the London Mathematical Society. During his brief lifetime, he published but one book and various papers based on his college lectures. His work has since been reconstructed and edited, perhaps the most popular item being this edition by Karl Pearson.

IT was in 1875, when Clifford was in fairly good health, that he dictated the whole of three chapters and part of another for a projected book to be entitled 'The First Principles of the Mathematical Sciences Explained to the Non-Mathematical.' Three years later, shortly before his death, he expressed the wish that the book should be published only after very careful revision, and that the title should be changed. It has certainly not received the sort of revision that Clifford desired; for as published it abounds in errors, and contains several quite anti-Cliffordian views. For instance, he says that if a point on the surface of a sphere is brought into contact with a point on the flat face of a cube, "we cannot move the sphere ever so little without separating these points." This is erroneous, because we can spin the surface about the point of contact; but although the passage has passed under the hands of two successive mathematical editors, neither has seen, what the course of reasoning shows, that Clifford in dictating said "move" when he meant *roll*. He wanted to show that all surfaces would fit together at any points where they are not broken by edges or corners, much as a ball may fit into a cup, only that the fitting is confined to a single point. Now surfaces that fit together may or may not be capable of being slipped or spun one on the other, but they cannot be rolled one on the other. A rolling motion, therefore, was the only one which had to be considered. Again, he defines a surface as the boundary between two portions of space which it separates absolutely. Now, without speaking of spirals, which obviously do not separate space into two parts, the most familiar of all surfaces, the plane, does not do so (according to the conception of the modern geometrician). Two planes will separate space, and one of these may be the plane at infinity; but a single plane does not. For if a point (say the focal point of a lens) be carried off with sufficient acceleration from one side of a plane, it will come back on the other side. Every surface may, it is true, form a part of the boundary between two regions of space. But even so modified, the definition is hardly satisfactory; for the calculus requires us to suppose that a solid body may approach indefinitely near to being a surface, which it certainly could not do were the two objects essentially disparate in their nature. Clifford here says:

"The surface of a thing is something that we constantly observe. We see it and feel it, and it is a mere common-sense observation to say that this surface is common to the thing itself and to the space surrounding it." "The important thing to notice is that we are not here talking of ideas or imaginary conceptions, but only making common-sense observations about matters of every-day experience."

But, as the editor, "K. P.," remarks, "we are compelled to consider the surface of the geometer as an idea or imaginary conception, drawn from the apparent (not real) boundaries of physical objects." The truth is, that the geometrical conception of space itself is a fiction. The geometer thinks of space as an individual thing or (as Mr. F. E. Abbot expresses it) a receptacle of things having an existence as something individual. If this were so, absolute position in space (independent of other bodies) and absolute velocity would have a meaning; but, in fact, they appear to have none. What is true is, that rigid bodies in their displacements are subject to certain laws which are the principles of geometry; and we have an instinctive acquaintance with these positional laws, which makes it easy for us to imagine the fictitious receptacle in which these laws are embodied. Thus, space only exists under the form of general laws of position; there is really nothing individual about it. And easy as is the geometer's conception, it is by no means born in us. The natural man knows of space only as a synonym for "air." Kant is responsible for the perpetuation of the erroneous conception of space which Leibnitz had escaped. It is impossible to have clear ideas concerning the non-Euclidean geometry, space of *n* dimensions, and such matters, without a proper understanding of this.

The main fault of the whole plan of the book is, that while it gives no adequate explanation of many mathematical conceptions interesting to a large body of non-mathematical minds—such as the square root of the negative, multiple algebra, space of n dimensions, the mathematical conception of the Absolute, non-Euclidean space, invariants, Riemann's surfaces, etc., conceptions perfectly susceptible of clear and interesting explanation, without too severely taxing the powers of the non-mathematical—it does suppose a reader whose interest in the logical *enchaînement* of mathematics is exceptionally great. Nine persons out of ten will read the chapter on number and exclaim, "This is nothing but what we learned at school," thus missing the whole argument, which will fly over their heads unperceived. The book has something of Clifford's style and traces of his power, but only faint ones. It will be of some service, but not very much. The parts added by "K. P.," one chapter and a half, bear comparison with those written by Clifford; it is a pity that the revision of the latter has not been more minute and accurate.

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The Religion of Philosophy; or, The Unification of Knowledge: A comparison of the chief philosophical and religious systems of the world, made with a view to reducing the categories of thought, or the most general terms of existence, to a single principle, thereby establishing a true conception of God.

By Raymond S. Perrin. G. P. Putnam's Sons. 1885.

CSP, identification: MS 1370. See also: Fisch, *First Supplement*. This note is unassigned in Haskell's *Index to* The Nation, vol. 1.