The English language fails to discriminate in precise terms between law in the abstract, and a particular law, defined by the political sovereign, which are represented in most other languages by distinct terms: e.g. "ius" and "iuris". "Right" and "law" prevent these notions inadequately, because although generally the former was used by the Anglo-Saxons like the German "recht", as in tolerand, with us it now includes the whole domain of morals.

LAW—LAWS OF THOUGHT

It is noticeable that two of those propositions are categorical and the third disjunctive; a circumstance demanding explanation for those who hold the distinction of material, conditional, and disjunctive propositions to be fundamental.

The meaning of the formula of identity presents only one difficulty. If the copula "is" be taken in the sense of "is, if it exists," then the meaning of the formula is that no universal affirmative proposition having the same term in subject and predicate is false. If, however, the copula be understood to imply existence, the meaning is that no universal affirmative proposition is false in which the same term in subject and predicate is involved. In such a formula it denotes any existing object. Or, the meaning may be that the same thing is true when the subject and predicate are the same proper name of an individual. In any case, it may properly be required that the preceding meaning attached to the copula should be explained; and this explanation must in substance involve one or other of the above three statements; so that in any case the principle of identity is merely a part of the definition of the copula.

In the latter, if the word "is" is to be used in logical forms, its force should be explained with the utmost precision. Such no explanation will consist in showing that the relation it expresses belongs to some certain classes of relations, probably not more than two or three, in view of the simplicity of the idea.

Each of these two states may be considered in a formula similar to a general way, as to the form of contraposition and excluded middle. It has, therefore, to Mill and to the "exact" logicians that these two formulas ought together to constitute a definition of the form "is, not." Other writers have regarded all three as constituting the third of a perfectly general law of thought, a law which may be said to have "validity" or "truth" but not to be "true". But it is only when they are formally stated, that it is seen that they are not all true.

1. The Principle of Identity: A is A.
2. The Principle of Contraposition: A is not not A.
3. The Principle of Excluded Middle or Excluded Third: everything is either A or not A.
Laws of Thought

Considered as part of a definition, one of the commonest statements of the principle of contradiction is: "A is not non-A," or: "Nothing is both A and non-A." The principle of excluded middle is: "What is not A is B." If, however, we prefer to state the principle of excluded middle as: "Every term is either A or non-A," then we should state the principle of contradiction as: "What is not A must be non-A." In either case, A and non-A must be either one or another, or some equivalent.

Thus, we may express the principle of contradiction as follows:

Whatsoever there may be which is both A and non-A is not A, which is no matter what term A may be.

In either formula, A may be understood to be restricted to being an individual, or it may be allowed to be any term, individual or general. In the former case, in order to avoid conflict with the fundamental law that no contradiction can exist, a special clause should be added, such as: "If not-A be not-b." In the latter case, it should also be added that: "Not-A is not not-B." Not only must A not be non-A, but there must be a non-A also. The principle of contradiction establishes the relation of "not" to all of the forms of A.

The principle of excluded middle makes the relation of "not" to include all pairs of the form A and B.

From this point of view, we see at once that there are three other similar pairs of forms defining the relations of identity, coincidence, and inconsistency in the following:

Whatsoever is B is similar to whatever is A, which is the same as the following:

Every man is loved by whatever loves whatever is a man.

But if we turn to the corresponding forms of the excluded middle, "Not-A is not A," or: "Not-A is not anything that is not A," we find that its meaning cannot be so simply expressed. Supposing that the relation is thus that it is true that whatever is A to whatever is B to whatever is C, etc.

Much has been written concerning the relations of the three principles to forms of syllogism. They have, as usual, been called, the three principles of St. Thomas, and there have been a great number of syllogisms in this subject, which have been in many different forms, either in the other, or in the one, or in the other. They have been divided into two main classes, according to the number of terms in each syllogism: the former being those in which only one term is used, and the latter those in which two or more terms are used. These are the principal syllogisms in the system of St. Thomas, and they are: the syllogism of St. Thomas, the syllogism of St. Thomas and St. Thomas, and the syllogism of St. Thomas and St. Thomas and St. Thomas. The syllogism of St. Thomas is: "A is B, and B is C, therefore A is C." The syllogism of St. Thomas and St. Thomas is: "A is B, and B is C, therefore A is C." The syllogism of St. Thomas and St. Thomas and St. Thomas is: "A is B, and B is C, therefore A is C." The syllogism of St. Thomas and St. Thomas is: "A is B, and B is C, therefore A is C."
all of that which is other than a—that is, if everything must be one or the other (a or b) and if nothing can be both. These two properties constitute the definition of a pair of contradictories (whether terms or propositions), namely, they are mutually exclusive and they are together exhaustive; expressed in the language of ‘exact logic,’ these properties are (writing # for the negative of a and + for or):

(1) a < a,
(2) a + b, 
what is at once a and b does not exist, or, in the language of propositions, what can be the conjoint occurrence of a and b does not take place.

Together these properties constitute the requirements of contradiction or of exact negation; it is a very inadequate piece of nomenclature (besides that it leads to actual conclusion) to refer to (1) alone as the principle of contradiction. Better names for them are (1) exclusion and (2) exhaustion (in place of excluded middle). In the common phraseology we are obliged to commit the absurdity of saying that two terms or propositions may satisfy the ‘principle of contradiction’ and still not be contradictory since they may lack the quality of being exhaustive. The mere fact that (1) has been called the principle of contradiction has given it a pretended superiority over the other which it by no means possesses; they are of equal importance in the conducting of reasoning processes. In fact, for every formal argument which rests upon (1) there is a corresponding argument which rests upon (2); thus in the case of the fundamental law of Transposition (q.v.) which affirms the identity of these two propositions, (m) the student who is not a citizen is not a voter; (v) every student is either a citizen or a voter; that (m) follows from (v) depends upon one of these principles, and that (v) follows from (m) depends upon the other. These two names, exclusion and exhaustion, have the great advantage that they express the formation of subject.

Thus we may say that the test for the contradiction of two terms or propositions which are not on their face the negatives one of another is that they should be (1) mutually exclusive and (2) together exhaustive.

It may be noticed that if two terms are exhaustive but not exclusive, their negatives are exclusive but not exhaustive. Thus within the field of number, ‘prime’ and ‘even’ are exclusive (no number can be both) but not exhaustive (except in the limiting case of two, some numbers can be neither), while ‘not even’ and ‘not prime’ are exhaustive and not exclusive.

In the case of propositions, ‘contrary’ and ‘subcontrary’ are loosely chosen names for the proposition (q.v.) of A and B, O and L, respectively, of the traditional logical scheme; they do not carry their meaning on their face, and hence are unnecessarily difficult for the learner to bear in mind. A and B should be used to be mutually exclusive (but not exhaustive), O and L to be completely exhaustive (but not exclusive).—This relation of qualities is then seen to be a particular case merely of the above-stated general rule.

Again, ‘no a is b’ and ‘all a is b’ are exclusive but not exhaustive, while ‘some a is b’ and ‘some a is not b’ are exhaustive but not exclusive (provided in both cases that a exists).

Laws of thought is not a good name for these two-characteristics; they should rather be called the laws (if laws at all) of negation. Properly speaking, the laws of thought are all the rules of logic; of these laws there is one which is of far more fundamental importance than those usually referred to under the name, namely, the law that if a is b and b is c, it can be concluded that a is c. This is the great law of thought, and everything else is of minor importance in comparison with it. It is singular that it is not usually enumerated under that name. Another law of thought of equal consequence with those usually so called is, according to Sigerist, the law that the double negation is equivalent to an affirmative.

\[ \neg \neg a \equiv a \]

But these are not fundamental, for from the principles of

<table>
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<th>Exclusion</th>
<th>Exhaustion</th>
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<tr>
<td>a &amp; \neg a</td>
<td>\neg a &amp; a</td>
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it follows by (c) that

\[ \neg a \Rightarrow \neg \neg a \]

(C.L.F.)

Literature: for the history of these principles see Ueberweg, Syst. d. Logik, §§ 15–40; Prantl, Gesch. d. Logik (see ‘principles’ in the indices to the four volumes). There are additional notes in an appendix to Hamilton, Lects. on Logic.

END OF VOL. I.