CRITICISMS AND DISCUSSIONS.

SUBSTITUTION IN LOGIC.

To the Editor of The Monist.

In the Journal of Philosophy, Psychology, and Scientific Methods (Vol. I, p. 541) Professor James, the eminent Harvard psychologist, makes the following positive assertion:

"In Taine's brilliant book on 'Intelligence,' substitution was for the first time named as a cardinal logical function, though of course the facts had always been familiar enough."

Now I should like to put this question to your readers: Are not the statements contained in the following sentences what may fairly be called "the naming of substitution as a cardinal logical function"?

"Every conclusion may be regarded as a statement substituted, for either of its premises, the substitution being justified by the other premises. Nothing is relevant to the other premises except what is requisite to justify this substitution. Every substitution of one proposition for another must consist in the substitution of term for term. Such substitution can be justified only so far as the first term represents what is represented by the second."

These sentences occur in a pamphlet entitled Three Papers on Logic, by C. S. Peirce, which was, as I am informed, widely distributed in the summer of 1869. The same papers were also printed early in 1868 in Vol. VII of the Proceedings of the American Academy of Arts and Sciences, pp. 259 to 298, as having been presented to that society in March, April, and May, 1867. Taine's work De l'intelligence is dated; its preface, December, 1869.

Since Ockham, Hobbes, and Leibnitz, who all regarded mind from the same general standpoint as Taine, like him, spoke of thoughts as signs substituted for things and for other signs, the question as to whether or not any great step in logic was made in thus regarding substitution as the "cardinal function," is one of too large a scope to be here entered upon; but I subjoin a few more sentences from the papers referred to to show that the conception was not left undeveloped by Mr. Peirce.

"The objects of the understanding, considered as representations, are symbols, that is, signs that are at least potentially general. But the rules of logic hold good of any symbols, of those that are written or spoken, as well as of those that are thought."

"Symbols which directly determine only their imputed qualities are but sums of marks, or terms;"

"Symbols which further independently determine their objects by means of either term or terms, and thus, expressing their own objective validity, become capable of truth or falsehood, are propositions;"

"Symbols which still further independently determine their interpreters, and thus the minds to which they appeal, by premising a proposition or propositions which such a mind is to admit, are arguments."

Mr. Peirce seems to have regarded it as essential to an argument that it should appeal to the interpreting mind to judge of it independently. Thus, he says, "an argument will here denote a body of premises considered as such, for it must distinctly show what the interpretation of the premises is expected to be, yet, in so far as the argument is a rational appeal, the conclusion which embodies this interpretation is not put as an assertion, but is only formulated and submitted to the interpreting mind to judge.

Mr. Peirce has always been careful to exclude from logic, matter that he considers psychological, and therefore it is not surprising that he did not explain to what mind the appeal of the argument is addressed when one reason with oneself. But it would seem to be plain from the above extracts, and is rendered perfectly clear in the papers referred to, that he not only considered all logical thought as an operation upon symbols consisting in substitution, but that he undertook to demonstrate this and to show how the same is true.

I may add that Peirce does not in the papers referred to say that substitution, which he makes the one hinge of all reasoning, is an indecomposable operation, and that in Baldwin's Dictionary of Philosophy and Psychology, Article "Symbolic Logic," he shows that no operation of substitution is valid unless the operations of insertion and subsequent omission into which it can be resolved are both valid.

-FRANCIS C. RUSSELL.

THE PLACE OF MATHEMATICS IN EDUCATION.

The present rector of the University of Munich, Professor Ferdinand Lindemann, has devoted his official rectorate to the important subject of the significance of mathematics in the higher schools. At present the curriculum of the German gymnasium is based upon the principle that education consists first of all in a knowledge of classical philology and history. Pro-